Sustainable growth and financial markets in a natural resource rich country

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Natural resource Curse (Sachs and Warner, 1997) = Countries with great natural resource (NR) wealth tend to grow more slowly than resource-poor countries

Natural resource rich countries (ex: oil, gaz) are usually indebted countries.

Ex: In 2000-2002, the public debt-GDP ratio is 78% for Gabon, 92% for Angola the 2nd oil producer of Africa
Introduction

Stylized facts

Figure: Growth and natural resource abundance (Real Growth per capita 1970-1989 and Exports of natural resources in percent of GDP)

Source: Sachs and Warner, 2001
Figure: Oil Production Forecasts in Azerbaijan

(In millions of barrels between 2003 and 2029)

Source: Center for Economic and Social Development, Baku, April 2011
Our paper is at the crossroads of two literatures:
- extensions of the Ramsey model to an open economy: Barro and Sala-i-Martin (2003)

Gap in the literature that focuses only on sustainable growth in closed economies.
Can a small open economy with non renewable NR have sustainable growth thanks to international borrowing ?

We introduce international borrowing in an exogenous Ramsey growth model with exhaustible non renewable natural resources.

- **1st step**: we consider a **constant interest rate**
- **2nd step**: we introduce a **debt-elastic interest rate** with constant natural resource prices, and then with increasing prices
The general model

- The production function:
  \[ Y = F(K, R) = K^\alpha ((1 - \gamma)R)^{1-\alpha}, \quad 0 < \alpha < 1 \]

- The man-made capital depreciates at rate \( \delta \):
  \[ \dot{K}(t) = I(t) - \delta K(t), \quad \delta \in [0, 1] \] (1)

- Natural resource sector
  \[ \dot{S}(t) = -R(t) \] (2)

with \( S \) the stock of natural resource (NR)
The intertemporal utility function is:

\[ \int_0^\infty e^{-\rho t} U(C(t)) \, dt \]

with \( U(C(t)) = \frac{C^{1-\eta}-1}{1-\eta} \) for \( \eta \neq 1, \eta > 0 \)
and \( U(C(t)) = \ln(C(t)) \) for \( \eta = 1 \)
The government’s dynamic budget constraint is:

\[ \dot{B}(t) = C(t) + rB(t) + I(t) - Y(t) - \gamma pR(t) \quad (3) \]
The general model

The government

\[
\max\{C, I, R\} \int_0^\infty e^{-\rho t} U(C(t)) \, dt
\]

s.t.

\[
\dot{K}(t) = I(t) - \delta K(t) \\
\dot{S}(t) = -R(t) \\
\dot{B}(t) = C(t) + rB(t) + I(t) - Y(t) - \gamma pR(t)
\]

\[K(0) > 0, S(0) > 0\]
The benchmark model with a constant interest rate

- From the optimality conditions, Marginal productivity of capital:
  \[ F_K = \delta + r \] (4)

- Marginal productivity of natural resources:
  \[ F_R + \gamma p = -\frac{\lambda_2(0)}{\lambda_3(0)}e^{rt} \] (5)

- Then the ratio capital natural resource is given by:
  \[ \frac{K}{(1-\gamma)R} = \left(\frac{\delta + r}{\alpha}\right)^{\frac{1}{1-\alpha}} \]
Proposition: The optimal rate of consumption is given by

\[ \frac{\dot{C}}{C} = \frac{r - \rho}{\eta}, \eta > 1 \]

As \( r \leq \rho \), the rate of consumption is negative, thus \( C \) is constant or

\[ \lim_{t \to +\infty} C = 0 \]

⇒ confirms the literature extending the Ramsey model to an open economy with international borrowing
The benchmark model with a constant interest rate

- **Proposition:** *The optimal path of output and stock of capital approach zero.*
- NR are exhaustible, so that the rate of extraction of those resources tends towards zero:
  \[
  \lim_{t \to +\infty} R = 0
  \]
- Since the ratio \( \frac{K}{R} \) is constant, the accumulation of capital also approaches zero:
  \[
  \lim_{t \to +\infty} K = 0; \quad \lim_{t \to +\infty} I = 0
  \]
  \[
  \lim_{t \to +\infty} Y = 0
  \]
  ⇒ refutes the literature extending the Ramsey model to an open economy with international borrowing
Counterfactual no-output and no-growth results

Therefore, in a small open economy with exhaustible NR and a constant interest rate, positive growth cannot be sustained in the long run.

Attempts to improve those results in the literature: by introducing a constraint on international borrowing or adjustment costs. We will do it by endogenizing the interest rate.
The model with a debt-elastic interest rate

- The interest rate $r(B)$ depends now on the level of debt: $r(B)$ rises when the country’s debt increases.

**Figure**: Interest rate $r(B)$ in function of the level of debt
**Proposition:** When the interest rate is exponential, the optimal level of debt decreases and output falls to zero in the long-run.

We refer to Schmitt-Grohe and Uribe (2003) debt-elastic interest-rate premium:

\[ r(B) = r^* + \psi (e^{B-D} - 1) \]

This expression implies \( r(B) > 0, \ r'(B) > 0 \)
The model with a debt-elastic interest rate

Constant prices

- Using the optimality conditions of the general model, the ratio capital to natural resource:

\[
\frac{K}{(1 - \gamma)R} = \left(\frac{\delta + r'(B).B + r(B)}{\alpha}\right)^{\frac{1}{\alpha-1}}
\]

- By reorganizing our equations, we can find the following autonomous differential equation:

\[
\dot{B} = (r'(B).B + r(B)) \times \frac{\alpha}{G(B) \times r'(B)}
\]

- As \( r(B) > 0 \) and \( r'(B) > 0 \), and \( G(B) < 0 \),
  \( \Rightarrow \dot{B} < 0 \)
  \( \Rightarrow \) the optimal level of debt \( B \) decreases with time.
The model with a debt-elastic interest rate

Constant prices

- As the level of debt $B$ is decreasing towards zero, the ratio $\frac{K}{(1-\gamma)R}$ tends asymptotically towards a constant.  
  ⇒ same counterfactual results from the benchmark model with a constant interest rate $r$
  ⇒ $K, I, Y$ decrease asymptotically towards zero.

- We calibrate the debt path in function of time when the interest rate is exponential (Figure 2)
  We set $\alpha = 0, 32$, $\delta = 0, 1$, $p = 1$, $\gamma = 0, 5$, $r^* = 0, 04$,
  $\psi = 0, 8$ and $D = 0, 7442$
The model with a debt-elastic interest rate
Constant prices

Figure: Debt pattern in function of time
The model with a debt-elastic interest rate

Constant prices

Proposition: The growth rate of consumption is given by:

\[ g_C = \frac{\dot{C}}{C} = \frac{r'(B)B + r(B) - \rho}{\eta}, \eta > 1 \]

During the transitional dynamics, consumption grows at a positive rate.

But in the long-run, as B tends towards zero, \( g_C \) finally declines.
The model with a debt-elastic interest rate

Figure: Consumption growth rate in function of time

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The model with a debt-elastic interest rate

Increasing prices

Figure: Oil prices from 2002 to 2009, in dollars per barrel
The model with a debt-elastic interest rate

Increasing prices

- We now assume prices to increase at a rate $\theta$, with:
  \[ p(t) = p(0)e^{\theta t} \]
  Ex: 2002-2008 oil price went from 20$ to 140$

- We reexpress our equations as the following non autonomous differential equation, as $p$ depends now on time:
  \[
  \dot{B} = \frac{\alpha (r'(B)B + r(B)) - \gamma \dot{p}}{G(B) \ast r'(B)}
  \]

As we know from below $r(B) > 0$, $r'(B) > 0$, $G(B) < 0$, $\gamma > 0$ and $\dot{p} > 0$

- Therefore, if $\alpha (r'(B)B + r(B)) > \gamma \dot{p}$ then $H_2(B) = \dot{B} < 0$
  \[ \Rightarrow \text{the optimal level of debt is still decreasing, even though prices are increasing.} \]
Figure: Commodity Prices and Public Debt: The Case of Oil Producers
(Debt ratios in percent of GDP, oil prices in dollar per billion of barrels)

Source: IMF
In the model with a constant interest rate, output and consumption growth are not sustainable.

In the debt elastic interest rate model:
- Consumption grows for a while during the transitional dynamics and then decreases in the long-run
- The level of debt decreases asymptotically to zero, so do the output and the accumulation of capital
Next steps to improve the model:
- Introducing decreasing returns to scales
- Endogenize the $\gamma$, the share of natural resources exported abroad

Ongoing and future research:
- Empirical work on sovereign default risk in emerging natural resource rich countries
- Impact of the variation of oil price on oil countries’ Credit Default Swaps (CDS)